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SHEARING STRESSES IN REINFORCED CONCRETE BEAMS

Showing the Advantages of
RIGID CONNECTION
of
WEB REINFORCEMENT

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H. S. ROGERS, B. Sc., C. E.

Formerly Assistant Professor of Structural Mechanics, State
University of Washington

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Value of Rigid Connection for Fire Protection

The value of Kahn trussed bars as fire protection is worthy of much emphasis. "A splendid example to illustrate this point is in the building of the Dayton Motor Car Company which suffered a severe fire in one story. Sufficient heat was developed to practically ruin the lower inch or so of concrete in the slab above. This destroyed all adhesion in the main tension members. However, upon testing it was found that the strength of that floor was not diminished in the slightest. This was undoubtedly due to the rigidly attached diagonals which eliminated the necessity for adhesion in the main tension members. Undoubtedly, if loose stirrups had been used in place of rigidly attached members the fire would have practically ruined the floor or at any rate greatly diminished its strength." (Proceedings of the National Association of Cement Users, Vol. VII, page 259.)

Shearing Stresses in Reinforced Concrete Beams and the Advantages of Rigid Connection of Web Members

Kahn Trussed Bars

(Illustrations and Properties on page 12.)

Since the inception of reinforced concrete construction various types and systems of beam and joist reinforcement have come and gone. Exclusive of the bent bar and stirrup system of construction to which all types of deformed and plain bars are applicable, only one system of reinforcing has been used extensively. This system is the Truscon System, which because of its economy and structural adequacy has been able to hold a place in the construction world while other patented systems have vanished. The one element of the Truscon System of bar reinforcement which has established its superiority is the Kahn trussed bar with its rigidly-attached inclined shear prongs.

Kahn trussed bars embody the elements of economy:

- (1) They save steel and reduce cost of material,
- (2) They save time and labor and reduce cost of construction,
- (3) They not only provide the most effective structural reinforcement but also insure the structure against the hazards of shock, fire and careless workmanship.

Requirements of Beam Reinforcement

To properly reinforce a restrained concrete beam, steel is required at the bottom throughout the span, in the web toward the ends and in the top over the support. The ideal reinforcement would be an integral unit which provides the exact amount of steel required by the design at each of these three places. Theoretically this is possible, but certain empirical considerations interfere. It has been found advisable to retain at least one-half of the mid-span steel in the bottom throughout the entire length of the beam and to avoid numerous small bars.

The two principal types of beam reinforcement have, therefore, come into general use. The bent bar and stirrup designs utilize the bent bars for top steel and the stirrups for web steel; the Truscon System utilizes nested Rib bars for top steel

and the prongs sheared from the main bottom bars where the steel area is no longer necessary in the bottom as web steel. In both systems the bent bars which are provided for top steel are effective as web steel in that portion of the beam where they pass from bottom to top. In the bent bar designs, however, it is impracticable to care for shear by bars bent from the bottom of the beam solely for that purpose, because to do so would require too much steel taken from the bottom of the beam and numerous bars of small cross sectional area. The shear steel is, therefore, provided by bent stirrups of small cross sectional area which can be supplied only at a price higher than base price and with a charge for bending.

Saving in Cost of Steel and of Handling

Truscon Steel Company estimates have repeatedly shown savings of from 15% to 20%. This results from the fact that the shear steel is supplied by the area of the Kahn trussed bars which is no longer needed for longitudinal reinforcement. Small-sized stirrup bars, for which extra charges for size and fabricating are made, are eliminated while the Kahn trussed bars are furnished at prevailing base price plus a differential of \$3.00 per ton.

In addition to the saving in material costs the use of Kahn trussed bars with the rigidly attached shear prongs and large cross sectional areas greatly reduces the number of bars to be shipped, sorted and placed. This saving alone amounts to a considerable figure on many jobs.

Stress Lines in a Reinforced Concrete Beam

The structural adequacy of the Kahn trussed bar may be shown by a theoretical analysis as well as by practical tests. A very general theoretical consideration conclusively shows the effectiveness of Kahn trussed bar prongs both in sectional area and anchorage. The lines of *stress direction* in a

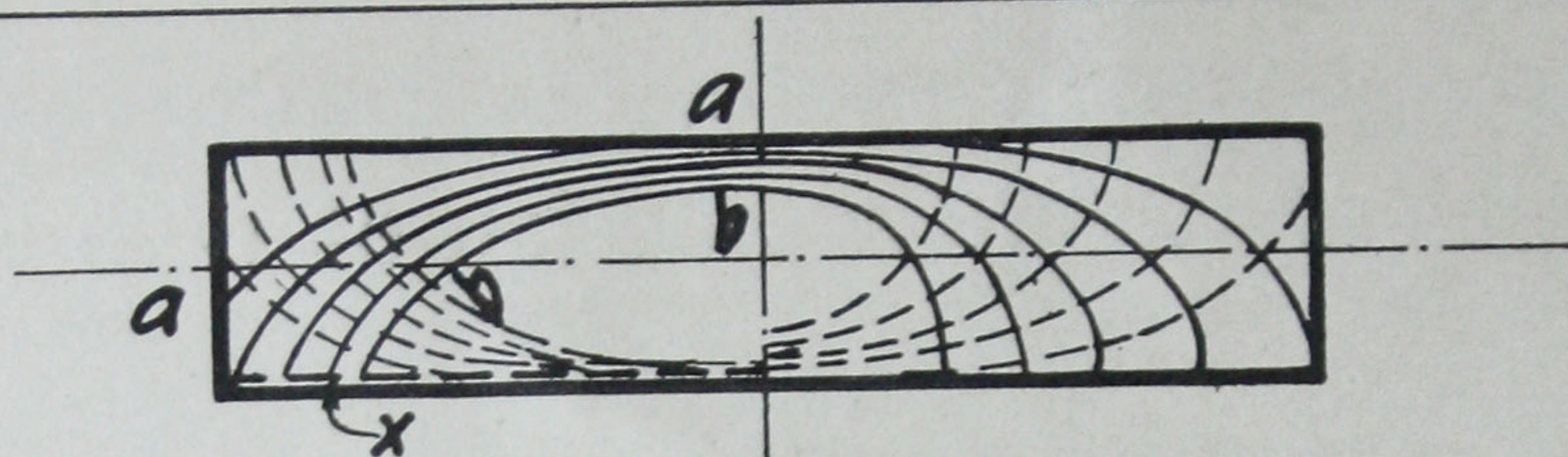


FIGURE 1

Showing general lines of stress direction in homogeneous and reinforced concrete beams.
 ———compression direction- - -tension direction.

homogeneous beam are shown on the right of Figure 1. The full lines are lines of compression and the broken lines are lines of tension. Tests have shown that in a reinforced concrete beam these lines will be modified. A greater portion of the tension will be carried along the steel, but it must be eventually carried to the compression in the concrete if it is to be neutralized by that compression. The direction of the stresses will be modified somewhat as shown on the left of Figure 1 and the values of the stresses will necessarily be changed with the change of direction. Tests by the United States Shipping Board corroborate this by showing a tension in the steel at "X" equal to as great a value as one-third¹ of that found in the steel at mid-span.

The results of these tests indicate that the action of a reinforced concrete beam lies somewhere between that of an arch bounded by the lines a-a and b-b of the figure with the steel tie rods across the bottom and that of a homogeneous beam. Under either condition of internal stress distribution the lines of compression and lines of tension are complementary. One set of stresses can not be changed without changing the other. This law is responsible for the high diagonal stresses reported in beams reinforced with loose stirrups. Such a distribution of stresses would indicate that the shear in a reinforced beam and the resultant diagonal stresses are carried by three agents (I) the concrete itself, (II) the longitudinal steel and (III) the web steel. Recent tests have shown that a "very considerable diagonal tension resistance is furnished by the concrete and longitudinal bars"² as well as by the steel bent into the web. The Kahn trussed bar is designed to work in harmony with this natural action in reinforced concrete beams.

Action of the Kahn Trussed Bar

(I) Because the shear prongs of Kahn trussed bars are bent up into a position at right angles to the compressive stresses in the concrete, these compressive stresses are reduced to a minimum and the tensile strength of the concrete at right angles to the compression is, therefore, the maximum. "The diagonal compression—in the concrete—is much higher where vertical stirrups are used than where inclined web bars are used."³ This increase in diagonal compression found in loose stirrup designs reduces the available diagonal tension strength at right angles to it and thereby restrains the concrete itself in its work of carrying diagonal stresses. The full strength of the concrete can only be reserved for useful work by placing the web steel in an inclined position.

(II) The tests carried out by the University of Illinois Engineering Experiment Station and the United States Shipping Board, which show that the action of a reinforced concrete beam is approximately that shown on the left half of Figure 1, prove that "where the longitudinal bars are anchored either by means of a half circular hook, of adequate size, by nuts and washers or other auxiliary device (*Kahn trussed bar prongs and lugs formed on main bar at ends of prongs*)"⁶ or by bond in the bar over or beyond the support, a considerably increased shearing resistance is secured."⁴ This increased shearing resistance is obviously secured by the rigid anchorage of the tie rods in the abutment of the arch of stress found in the concrete beam. The 1907 and 1908 series of tests at the University of Illinois showed this "increased shearing resistance" secured by anchorage of the longitudinal bar to be a maximum when the form of anchorage was rigid. The Kahn trussed bar prongs and lugs at points where

(1) Report of Special Committee on Unit Values for Vertical Shear in Reinforced Concrete Design, Page 6.

(2) Report of Special Committee on Unit Values for Vertical Shear in Reinforced Concrete Design, Page 6.

(3) Report of Special Committee on Unit Values for Vertical Shear in Reinforced Concrete Design, Page 5.

(4) Report of Special Committee on Unit Values for Vertical Shear in Reinforced Concrete Design, Page 4.

(6) Phrase in parentheses has been inserted.

prongs join the main bar form a most effective anchorage because of their rigid attachment, and thereby secure the maximum "shearing resistance" available through the action of the longitudinal steel.

(III) Theoretical analysis and tests prove that inclined web reinforcement is more effective than vertical, that rigidly attached stirrups are more certain provision against shearing forces than loose ones and that the amount of web steel per foot of longitudinal cross section of a beam can be quite accurately determined and should be variable, being a maximum at the support and diminishing toward mid-span. Kahn trussed bars provide inclined and rigidly attached stirrups which are adequate provision against usual shearing forces found in reinforced concrete construction. Kahn trussed bars with nested Rib bars—the Truscon System as now constructed—will provide sufficient shear reinforcement for the most extreme shearing forces. The inclined rigidly attached Kahn trussed bar prongs carry the tensile stresses in their natural direction from the longitudinal steel to the concrete where the compressive stresses neutralize them. They insure that there shall be no lost motion in taking the stress from the longitudinal bars into the web. This lost motion has frequently accompanied the ultimate collapse of beams in loose stirrup designs. They also insure the continued integrity of bond around the main bar. The illustrations given under the discussion of insurance against fire hazards are excellent examples of integrity of bond provided by Kahn bar prongs. Beams reinforced with Kahn trussed bars will possess greater strength, have less deflection at all loads and give gradual failure at ultimate loads.

Professor Talbot, commenting upon the results of web resistance offered by beams reinforced with longitudinal bars of which half were bent up into the web at an angle toward the ends, says: "The arrangement of bending up part of the bars and leaving part straight developed a good web strength and secured slow failures. This method will be particularly satisfactory when the number of bars is large enough to permit the spacing of the inclined bars at frequent intervals through the part of the

beam where high web resistance is needed. A proper distribution of the diagonals is important." ⁵ The Kahn bar prongs give this proper distribution.

Designing the Shear Members

The shear carried by Kahn bar prongs may be figured with the same degree of exactness and by the same formula that is universally used in determining web reinforcement. In its most recent form as given by the "Special Committee on Unit Values for vertical shear in Reinforced Concrete Design" of the American Concrete Institute the formula for designing web reinforcement is

$$A_v = \frac{V' a}{f_v j d}$$

In which A_v = The area of web member;

a = Spacing of web members perpendicular to their direction;

f_v = Stress in web member;

$j d$ = Lever arm of moment couple in beam;

V' = Shear carried by the steel.

In the Kahn trussed bar $a = d$ if the prongs are sheared opposite each other as shown in Figure 2.

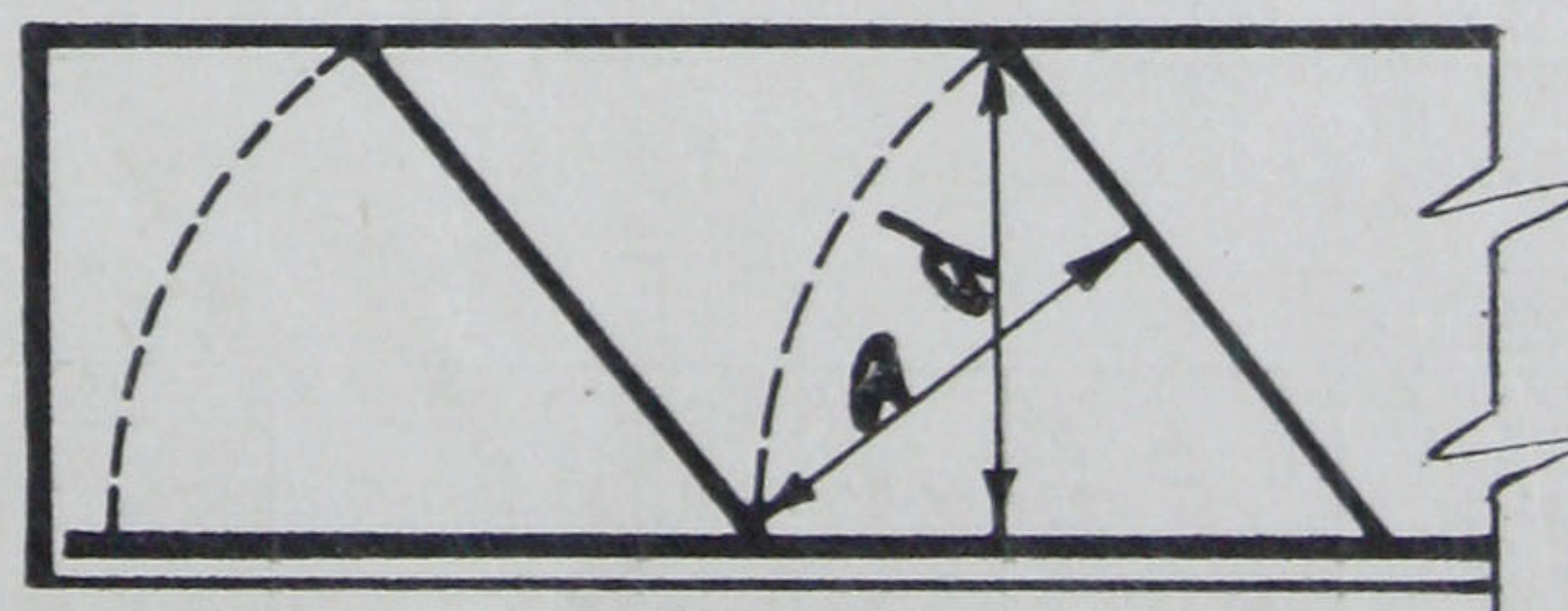


Figure 2

This is seen to be true regardless of the angle at which the prongs are bent, because each prong must be swung up as the radius of an arc and the perpendicular distances from corners of the arc to opposite radii are equal. The formula applied to the Kahn bar prongs, therefore, becomes

$$A_v = \frac{V' j}{f_v j}$$

Since "d" and "a" cancel from the equation and since the formula may be applied (as recommended

by the Committee) between all angles from 30 degrees with the horizontal to 90 degrees or vertical, the Kahn bar prongs may be considered as caring for a constant shear regardless of the angle at which they are bent, provided they are bent above 30 degrees and reach the top of the beam. Now if the prongs are staggered the perpendicular distance between prongs becomes $\frac{1}{2}d$ and the steel area becomes $\frac{1}{2}$ the original A_v ; so the amount of shear carried by the prongs is not changed but the steel is distributed better throughout the web.

The stress in Kahn trussed bar prongs when prongs carry the shear values given in "Design Tables" may be determined for any value of j . Table 1 gives the stress in prongs when the Kahn trussed bars carry the shear given in the design table and used in the Truscon System of design.

TABLE I

Kahn Trussed Bar	Gross Area	Net Area	A_v	V'	V'/j	f_v
$\frac{1}{2}" \times 1\frac{1}{2}"$	0.41	0.25	0.16	1800	2060	12800
$\frac{3}{4}" \times 2\frac{3}{8}"$	0.79	0.56	0.23	2600	2970	12900
$1\frac{1}{2}" \times 2\frac{1}{4}"$	1.41	1.10	0.31	3500	4000	12900
$1\frac{3}{4}" \times 2\frac{3}{4}"$	2.00	1.60	0.40	4600	5260	13100
$2" \times 3\frac{1}{2}"$	3.00	2.40	0.60	6800	7770	12900

The value of j used in the calculations is $\frac{7}{8}$.

$$f_v = \frac{V'}{A_v j}$$

A_v = (area of prongs) = (gross area of Kahn bar) - (net area).

Anchorage of Shear Members

The results displayed in Table I show that the stresses existing in Kahn bar prongs are well within the limits of conservative practice. The anchorage of the Kahn bar prongs, however, remains to be investigated.

Table II gives the computations for and values of embedment necessary to develop the prongs at a bond stress of 80 lbs. per sq. in. (See Appendix B.) If the stress in the prongs is to be developed in this embedment above the 0.6 depth plane of a beam as recommended in most conservative practice, the minimum length of prong, "L," can be obtained as follows:

$$\frac{L}{d} = \frac{\text{Embedment}}{0.6d}$$

$$L = \frac{\text{Embedment}}{0.6}$$

TABLE II

Kahn Trussed Bar	Gross Area	Net Area	Area Prongs	Thickness of Prongs	Perimeter of 2 Prongs	Stress Per Lin. In.	Stress in Prongs	Total Stress in Prongs	Embedment
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\frac{1}{2}" \times 1\frac{1}{2}"$	0.41	0.25	0.16	0.156"	2.675	214 lbs	12800	2060	9.6"
$\frac{3}{4}" \times 2\frac{3}{8}"$	0.79	0.56	0.23	0.203	3.077	246	12900	2970	12.1"
$1\frac{1}{2}" \times 2\frac{1}{4}"$	1.41	1.10	0.31	0.250	3.480	278	12900	4000	14.4"
$1\frac{3}{4}" \times 2\frac{3}{4}"$	2.00	1.60	0.40	0.250	4.200	336	13100	5260	15.6"
$2" \times 3\frac{1}{2}"$	3.00	2.40	0.60	0.344	4.851	388	12900	7770	20.0"

(4) Area of Prongs equals Area of two prongs.

(6) Perimeter of two prongs.

(7) Stress per lin. in. of two prongs developed by a bond of 80 lbs. per sq. in.

(8) Unit Stress in Prongs which they must carry to develop shear as given in design tables.

(9) Total stress carried by two prongs.

(10) Embedment of prongs necessary to develop the stress in prong which must exist when bars carry shear as given in the design tables.

Table III gives the minimum length of shear prongs necessary to develop the design stress in them by embedment above the plane of 0.6 depth, in a beam.

TABLE III

Kahn Trussed Bar	Min. Length of Prong
$\frac{1}{2}" \times 1\frac{1}{2}"$	16"
$\frac{3}{4}" \times 2\frac{3}{16}"$	20"
$1\frac{1}{2}" \times 2\frac{1}{4}"$	24"
$1\frac{3}{4}" \times 2\frac{3}{4}"$	26"
$2" \times 3\frac{1}{2}"$	33"

The values of the table show that the minimum lengths of shear prongs necessary are less (with one exception, that of the $\frac{1}{2}" \times 1\frac{1}{2}"$ bar) than those ordinarily used in the Truscon System. The $\frac{1}{2}" \times 1\frac{1}{2}"$ bar, however, is rarely used where it is necessary to develop its full shearing value, and furthermore theoretical analysis indicates that the full length of prong may be considered effective in developing the bar at point of juncture with the main bar. The Kahn trussed bar prongs closely follow the normal direction of tensile stresses in the beam and the stress in them is gradually removed by the compression in the concrete which grips the prong along its entire length. Now bearing in mind that the lines shown in the figure are lines of *direction of stresses* and that the stress diminishes in intensity along them, it is clear that a set of lines or contours of equal compressive and tensile stress exists at right angles to these and that the stress runs from both flanges of a beam to be neutralized by the stress of opposite kind just as water runs at right angles to the contours from any terrain. The necessary conclusion is that if the stress carried by a Kahn bar prong diminishes as normal action indicates it necessarily must, the bond carrying the stress from the prong must be effective throughout the entire length of the prong.

In all the reports of the tests upon beams containing longitudinal bars running straight throughout the beam with others bent into the web, there is no record of any case of slipping of bars bent into the web. That hooks in the compression flange upon properly proportioned inclined rigidly attached web members are essential is not proven and seems to be without foundation.

Tests of the University of Wisconsin

The most reliable data upon the adequacy of rigidly attached shear members and the relative

merits of loose stirrups and rigidly attached shear prongs is that presented in Bulletin No. 197 of the University of Wisconsin for the beams of series "N," in which series beams identical in all particulars of design excepting the shear steel are compared. The results of these tests as tabulated in Table XI on page 117 of the Bulletin are presented in TABLE IV.

TABLE IV

Tee Beam	Reinforcement	Max. Shear Lbs. per Sq. In.	Superiority of Kahn Trussed Bars over Loose Stirrups ⁷
N- 1	1, 1" square bar. 10, $\frac{7}{8} \times \frac{1}{4}$ Stirrups. . .	202	
N- 2	1, 1" x 3" K A H N TRUSSED BAR.	291	144%
N- 3	1, 1" Square bar. 12, $\frac{3}{8}$ " Stirrups.	218	
N- 4	1, 1" x 3" K A H N TRUSSED BAR.	314	144%
N- 5	2, $\frac{3}{4}" \times 2\frac{3}{16}"$ K A H N TRUSSED BARS.	200	136%
N-6	2, $\frac{3}{4}"$ Square bars. . . 14, $\frac{1}{4}"$ Inclined W Stirrups.	147	
N- 7	2, $\frac{3}{4}"$ Square bars. . . 20, $\frac{1}{4}"$ W Stirrups. . .	135	
N- 8	2, $\frac{3}{4}" \times 2\frac{3}{16}"$ K A H N TRUSSED BARS.	176	127%
N- 9	2, $\frac{3}{4}" \times 2\frac{3}{16}"$ K A H N TRUSSED BARS.	290	140%
N-10	2, $\frac{3}{4}"$ Square bars. . . 12, $\frac{1}{4}"$ Inclined W Stirrups.	207	
N-11	2, $\frac{3}{4}" \times 2\frac{3}{16}"$ K A H N TRUSSED BARS.	238	116%
N-12	2, $\frac{3}{4}"$ Square Bars. . . 16, $\frac{1}{4}"$ W Stirrups. . .	204	
	Average.		135%

The cross section of these beams is shown in Figure 3 on Page 8.

(7) These percentage results by ourselves.

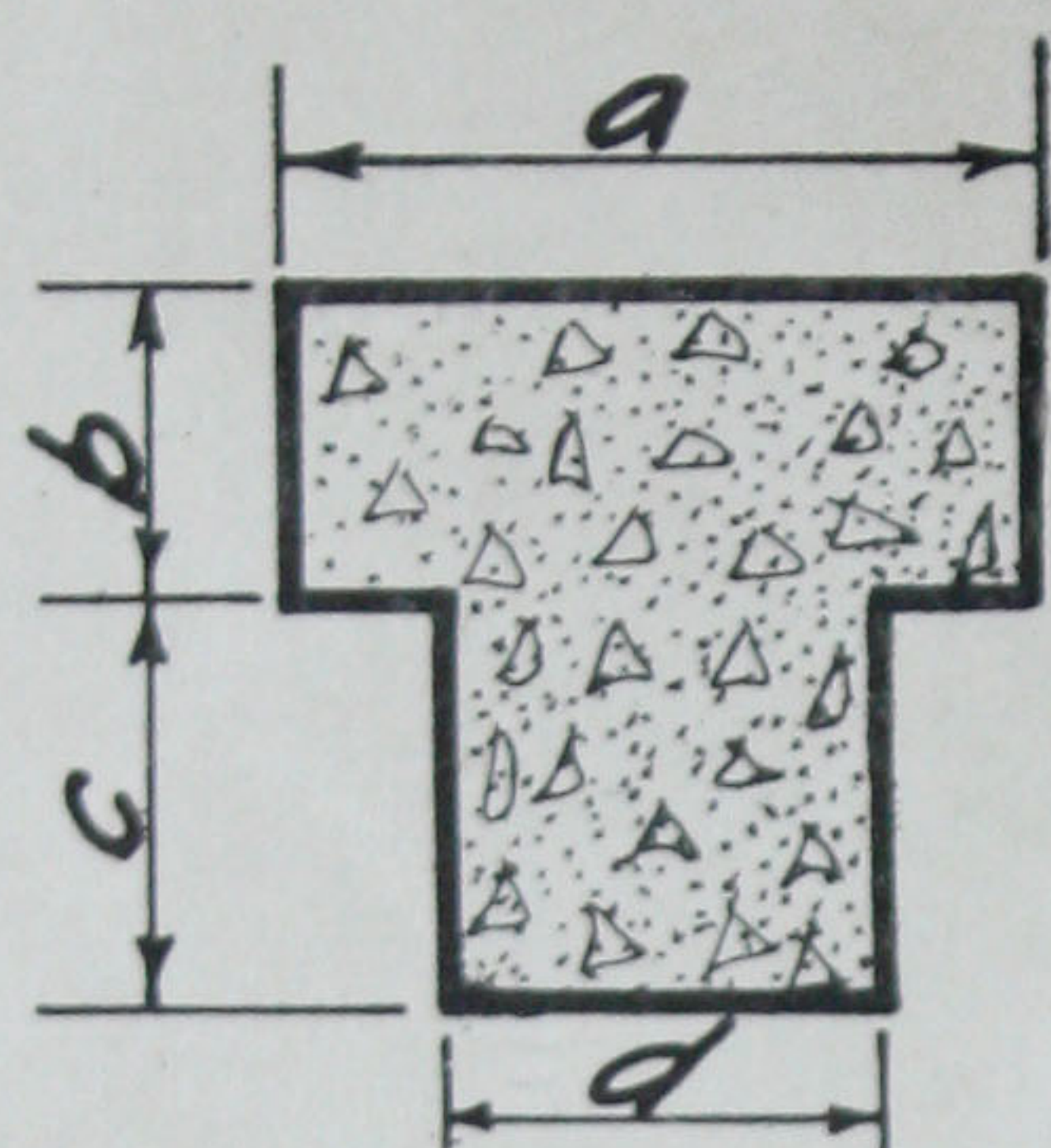


Figure 3

BEAMS	a	b	c	d
N1, N2, N3 and N4	12"	5"	8.65	5"
N5 to N12 inclusive	15 $\frac{1}{4}$ "	5"	6 $\frac{3}{4}$ "	8"

"In the beams with loosely attached web members and straight horizontal reinforcement, no rods bent up, the final failure occurred through slipping of the rods either before or just after the yield point of the metal was passed, when the load fell off rapidly. This did not occur in tests of the beams, with rigidly attached web members (Kahn bars). Beams N-1, N-3, N-6, N-7, N-10 and N-12 (*those reinforced with loose stirrups*) failed through slipping of the rods in the manner mentioned. Beams N-2, N-4, N-5, N-8, N-9 and N-11 (*those reinforced with Kahn bars*) failed in tension at the center. In those latter beams the steel was stressed considerably beyond the yield point and failure was very slow as shown by the deflection curves." ⁸ There was no failure by diagonal tension or by slipping of shear prongs. The results show an average shear carried by beams reinforced with Kahn bars equal to 135% of that carried by beams reinforced with loose stirrups and bars. This is certainly an overwhelming evidence of the efficacy of Kahn bars.

Value of Rigid Connection for Fire Protection

The value of Kahn trussed bars as fire protection is worthy of much emphasis. "A splendid example to illustrate this point is in the building of the Dayton Motor Car Company which suffered a severe fire in one story. Sufficient heat was developed to practically ruin the lower inch or so of concrete in the slab above. This destroyed all adhesion in the main tension members. However,

upon testing it was found that the strength of that floor was not diminished in the slightest. This was undoubtedly due to the rigidly attached diagonals which eliminated the necessity for adhesion in the main tension members. Undoubtedly, if loose stirrups had been used in place of rigidly attached members the fire would have practically ruined the floor or at any rate greatly diminished its strength." ⁹

Another example of the fire protection afforded by Kahn trussed bars is illustrated in the warehouse of Mullen & Buckley in Far Rockaway, N. Y. A fire and explosion completely stripped the fire-proofing from below the steel in several beams and girders supporting the fourth floor of the warehouse, and although the slab and one beam directly above the explosion chamber collapsed, the structure remained intact because the rigidly attached inclined Kahn trussed bar preserved the integrity of the other beams and girders. Several excellent photographs of the building showing the action of the Kahn trussed bars were reproduced in the Engineering News of November 16th, 1916.

The latter illustration indicates the value of Kahn bars in resisting shock and permits the inference that similar results may be expected where they are embodied in structures which have to undergo frequent repetition of stress or vibration.

Protection Against Careless Workmanship

The insurance against careless workmanship which Kahn trussed bars provide is produced by this same integrity of concrete and steel secured through the rigid attachment of stirrups. Eight of thirty-seven test beams described in detail in Bulletin No. 29 of the University of Illinois gave a lower average ultimate strength because the stirrups were exposed due to insufficient embedment. If such improper placement of stirrups can be found in eight of thirty-seven laboratory tests what accuracy of placement will be secured on the job?

That Kahn trussed bars do indeed embody all the elements of economy has been conclusively shown in thousands of designs. They do provide a most effective type of concrete reinforcement.

(8) Bulletin No. 197, University of Wisconsin, Page 114.

(9) Proceedings of the National Association of Cement Users, Volume VII, Page 259.

Appendix A

Anchorage of Web Reinforcement

In spite of the apparent and proved qualities of Kahn trussed bars it is sometimes necessary to defend them against a loose stirrup design which may be declared superior on the ground that the stirrups are hooked in the compression flange of the beam while the Kahn bar prongs are not thus securely anchored.

The anchorage of loose stirrups in the compression flange by means of hooks, may or may not be an essential feature in the design of beams to resist shearing stresses, but taken either way it does not necessarily follow that such anchorage in the compression flange is or is not essential at the ends of rigidly attached inclined shear prongs.

In view of the fact, however, that inclined web reinforcement has by analysis and test proven itself to be equal to or better than vertical reinforcement, the conclusion that anchorage is unessential in vertical reinforcement would justify a similar conclusion for inclined reinforcement.

The most extensive tests upon the shear resistance of beams reinforced with stirrups are those of the 1907 and 1908 series reported in Bulletin No. 29 of the University of Illinois Engineering Experiment Station, and many of the practices in reinforced concrete design are substantiated by these tests. In Bulletin No. 29 Professor Talbot says the "bond resistance of stirrups may be considered a critical stress." (Pages 72 and 73), but he also says (Page 73), "stirrups do not come into action, at least not to any great extent, until a diagonal crack has formed." The effectiveness of the stirrup hook is, therefore, in question until the beam is approaching ultimate collapse.

Thirty-seven beam failures of the 1907 and 1908 "Tests of beams with stirrups" are described in sufficient detail to permit analysis. Of these thirty-seven, thirty reinforced with stirrups which were not hooked made of a 1:2:4 concrete mix and containing various percentages of longitudinal steel between limits of 0.98% and 1.65% can be classified as to failure into five groups. The remaining seven can not be classified because of variation in concrete mix or lack of detailed description concerning failure.

GROUP 1 includes those failures due to tension in the steel,

GROUP 2 includes those failures due to diagonal tension in which the stirrup broke.

GROUP 3 includes those failures due to diagonal tension which were not directly traceable to failure of stirrups by slipping or in tension.

GROUP 4 includes those failures due to diagonal tension accompanied by stirrups slipping.

GROUP 5 includes those failures due to diagonal tension accompanied by stirrups breaking out because of improper placement and insufficient embedment or concrete breaking away from bottom of stirrups and those failures occurring in beams in which some of the stirrups were exposed upon the surface of the beams.

The groups with the beam numbers and unit shearing values together with the average unit shearing values are given in following Table.

Test Results of Beams with Stirrups

GROUP 1—Steel tension failure—four beams.

Beam Number	Max. Shear
232.1	281 lbs. per sq. in.
232.2	306 lbs. per sq. in.
233.1	328 lbs. per sq. in.
229.2	292 lbs. per sq. in.
$1207 \div 4 = 302$ lbs. per sq. in.	

GROUP 2—Diagonal tension—stirrups broke—two beams.

Beam Number	Max. Shear
231.1	251 lbs. per sq. in.
235.5	52 lbs. per sq. in.—poor concrete (1:5:10 mix.)*

GROUP 3—Diagonal tension failure—seven beams.

Beam Number	Max. Shear
232.6	192 lbs. per sq. in.
243.1	336 lbs. per sq. in.
223.1	146 lbs. per sq. in.
223.6	162 lbs. per sq. in.
225.5	173 lbs. per sq. in.
229.5	159 lbs. per sq. in.
223.4	201 lbs. per sq. in.
$1369 \div 7 = 195$ lbs. per sq. in.	

* All beams excepting this one were of a 1:2:4 mix.

GROUP 4—Diagonal tension failure—stirrups slipped—nine beams.

Beam Number	Max. Shear
233.7	208 lbs. per sq. in.
222.5	174 lbs. per sq. in.
224.5	168 lbs. per sq. in.
221.3	242 lbs. per sq. in.
221.4	166 lbs. per sq. in.
223.2	189 lbs. per sq. in.
223.3	201 lbs. per sq. in.
229.1	255 lbs. per sq. in.
243.3	260 lbs. per sq. in.
$1863 \div 9 = 207 \text{ lbs. per sq. in.}$	

GROUP 5—Diagonal tension failure—stirrups exposed—eight beams.

Beam Number	Max. Shear
233.3	264 lbs. per sq. in.
221.2	203 lbs. per sq. in.
222.5	182 lbs. per sq. in.
225.7	165 lbs. per sq. in.
227.5	158 lbs. per sq. in.
227.6	130 lbs. per sq. in.
228.5	138 lbs. per sq. in.
228.6	168 lbs. per sq. in.
$1408 \div 8 = 176 \text{ lbs. per sq. in.}$	

Certain facts are patent

(1) Stirrups without hooks seem to have cared for or assisted in caring for some of the highest unit shearing stresses obtained in the tests, viz., those of Group 1.

(2) Stirrups without hooks developed sufficient anchorage to stress them to ultimate collapse as shown in Group 2.

(3) Beam containing stirrups which did not slip before or at the ultimate failures of the beams, carried on the average lower unit shearing stresses than beams containing stirrups which did slip at or before failure as shown by Groups 3 and 4. Certain questions at once arise. Was the slipping of the stirrups in Group 4 the cause of ultimate failure or was it just a phenomenon accompanying the breaking up of the beams? How effective would hooks have been in preventing such failures? The comments in the Bulletin on these failures all contain such remarks as, "Diagonal crack marked (4) opened and extended to load point." "Failure crack was more nearly vertical," or "Two stirrups marked (a) slipped as crack (2) opened." An examination of the illustrations showing "Disposi-

tion of Reinforcement and Position of Cracks" shows that the cracks run diagonally through the web up into the compression flange to the top of the beam or vertically along the plane of the stirrups in all cases. Will hooks prevent such failures? The "Report of Special Committee on Unit Values for Vertical Shear in Reinforced Concrete Design" presented to the American Concrete Institute in February, 1920, contains on Page 6 the remark: "Recent tests show clearly that it is unsafe to use bars whose diameter exceeds $1/40$ to $1/50$ of the beam depth, for even with stirrups anchored on other steel at each end, such bars slipped in some cases before developing a yield point stress. A $3/4$ " diameter bar was too large for a beam 36" deep when placed vertically and a 1" bar was too large for the same depth when placed on a 45 degrees slope." Just how effective then are hooks on stirrups? Conservative practice would dictate that they be used, but tests indicate that the embedment of the stirrup in the concrete enables it to impart its stress to the concrete throughout its length instead of at the point of the hooked anchorage, and thus makes it an effective form of shear reinforcement. It is significant that the values $1/40$ to $1/50$ of the beam depth for the diameter of stirrups recommended by the Committee will give the values of embedment ordinarily used in design to develop bars in tension, i. e., 40 diameter embedment to develop a corrugated bar which produces an average bond stress of 100 lbs. per sq. in. and 50 diameter embedment to develop a plain round which produces an average bond stress of 80 lbs. per sq. in. These values of embedment in the tension flange of a beam have been substantiated by tests of the "United States Shipping Board" as shown in Appendix B.

One more important fact remains to be pointed out; it is that indicated by the tests of Group 5, viz., that eight of twenty-five beams of a 1:2:4 mix failed by diagonal tension gave clear evidence of poor workmanship and failed at the lowest average unit stress of the groups. The workmanship was that of an experimental laboratory; what is to be expected "out on the job" where the necessity of proper embedment is not realized?

It may be pointed out that beams No. 241.5 and No. 241.6 of the 1907 series which were reinforced with hooked stirrups and which contained the only hooked stirrups in the tests gave values of unit shear equal to 246 lbs. per sq. in. and 224 lbs. per

sq. in. These values are greater than the average values given by the groups of beams which failed in diagonal tension. At the same time, however, it may be pointed out that beams No. 231.5 and No. 231.6 containing the same brand and mix of cement, the same percentage and type of reinforcement, and stirrups which were not hooked gave values of unit

shear equal to 272 lbs. per sq. in. and 242 lbs. per sq. in., respectively. That a condition may be pictured in which hooks on stirrups may delay the ultimate failure of a beam is without question, but a claim of superiority of loose stirrups based upon hooks as contrasted with rigidly attached inclined shear members without hooks is not substantiated.

Appendix B

Bond Stress

In determining the length of embedment necessary to develop the prongs of Kahn Trussed bars a bond stress of eighty pounds per square inch has been used. This has been justified by the results of several years experience. Elaborate investigation of the internal stresses in concrete were made during the late war by the Concrete Ship Section of the Emergency Fleet Corporation and the results obtained have recently been published.

In a paper describing some of these investigations read before the American Concrete Institute, Mr. W. A. Slater shows conclusively that "the length of lap required" is a function of the diameter of and the stress to be developed in the bar. Tests on bond stress were made on reinforced concrete beams in which the bars were arranged as shown in Figure "X". The beams were loaded at the third points and, therefore, the bending moment varied as shown in Figure "Y."

The results of tests carried on by United States Shipping Board were expressed in the formula:

$$L = 0.00118 a f_s^*$$

in which L = length of embedment in inches from end of bar to point where bond stress becomes zero.

a = diameter of bar.

f_s = the tensile unit stress under consideration.

Using a yield point stress for f_s equal to 32,000 lbs. per sq. in. because the same factor of safety should exist in bars embedded in the tension flange as exists in all bars used in reinforcing concrete, the equation gives

$$L = 0.00118 a \times 32000 = 37.76 a$$

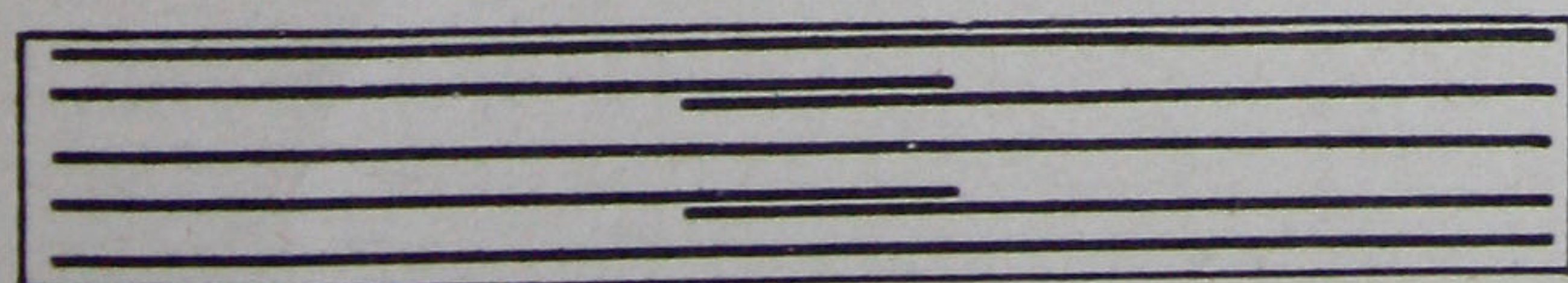


Figure "X," Plan View

This embedment would be necessary to develop a bar in the tension side of a concrete beam made of a quality of concrete equal to that of the concrete used in the tests.

The mix of the concrete used was $(1:\frac{2}{3}:1\frac{1}{3})$ and the pull out values of bond stress obtained at an end slip of 0.01" which is usually assumed to be the slip at maximum bond resistance was 600 lbs. per sq. in. as reported in the same paper. Assuming the bond stress at working values in different concretes, varies as the bond stress at 0.01" slip in the same concrete and taking a bond stress of 450 lbs. per sq. in. as that produced by a pull out test in 1:2:4 concrete, a value which is shown to be fair in tests described in Bulletin No. 71 of the University of Illinois Engineering Experiment Station, the necessary embedment should be

$$L = \frac{600}{450} \times 37.76 a = 50.35 a$$

Taking a length of embedment of 50 diameters and a working stress in the bar of 16,000 pounds per square inch the average bond stress can be found as follows:

$$U = \frac{16,000 \times a^2}{50 a \times 4 a} = \frac{16,000}{200} = 80 \text{ lbs. per sq. in.}$$

This is true whether the bars are round or square. This demonstrates that in using a bond stress of 80 lbs. per sq. in. to determine the necessary length of embedment of Kahn bar prongs sound practice has been followed.

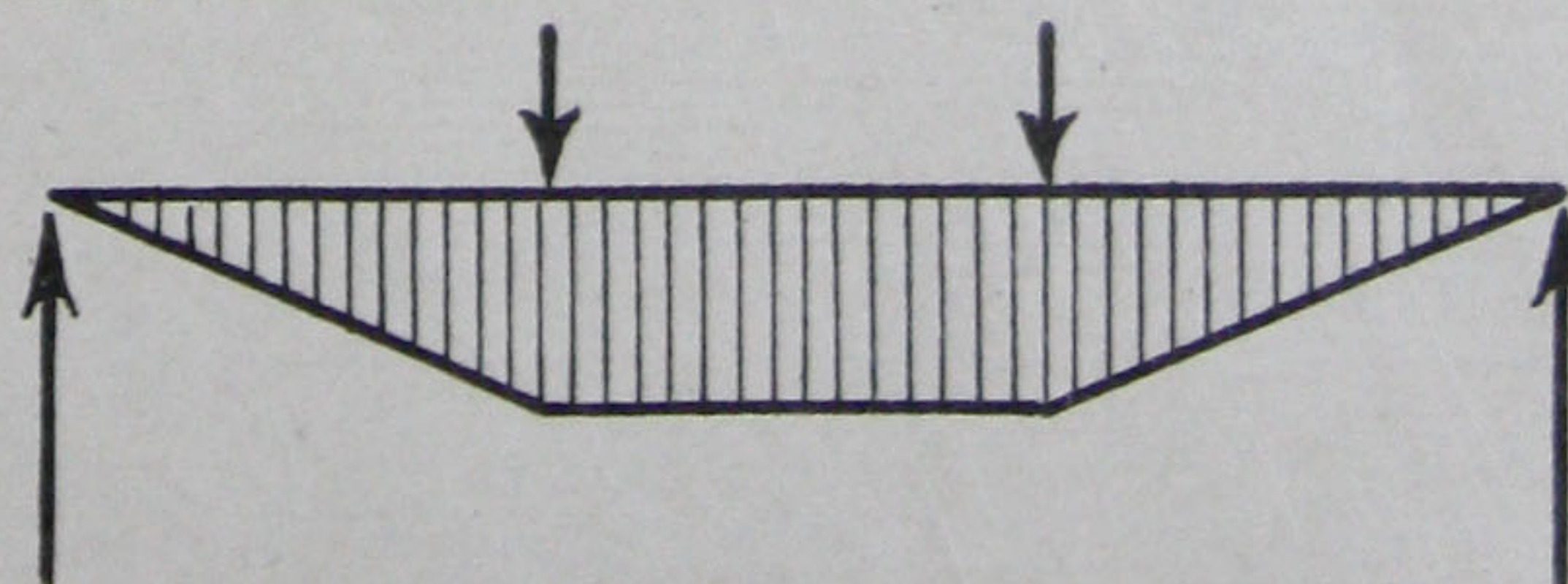
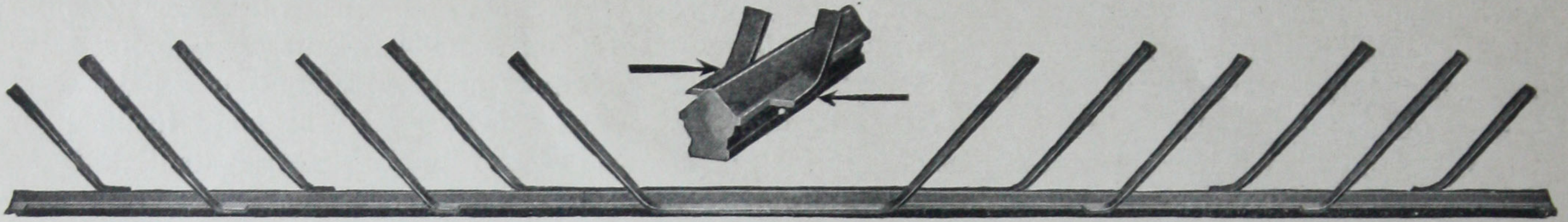


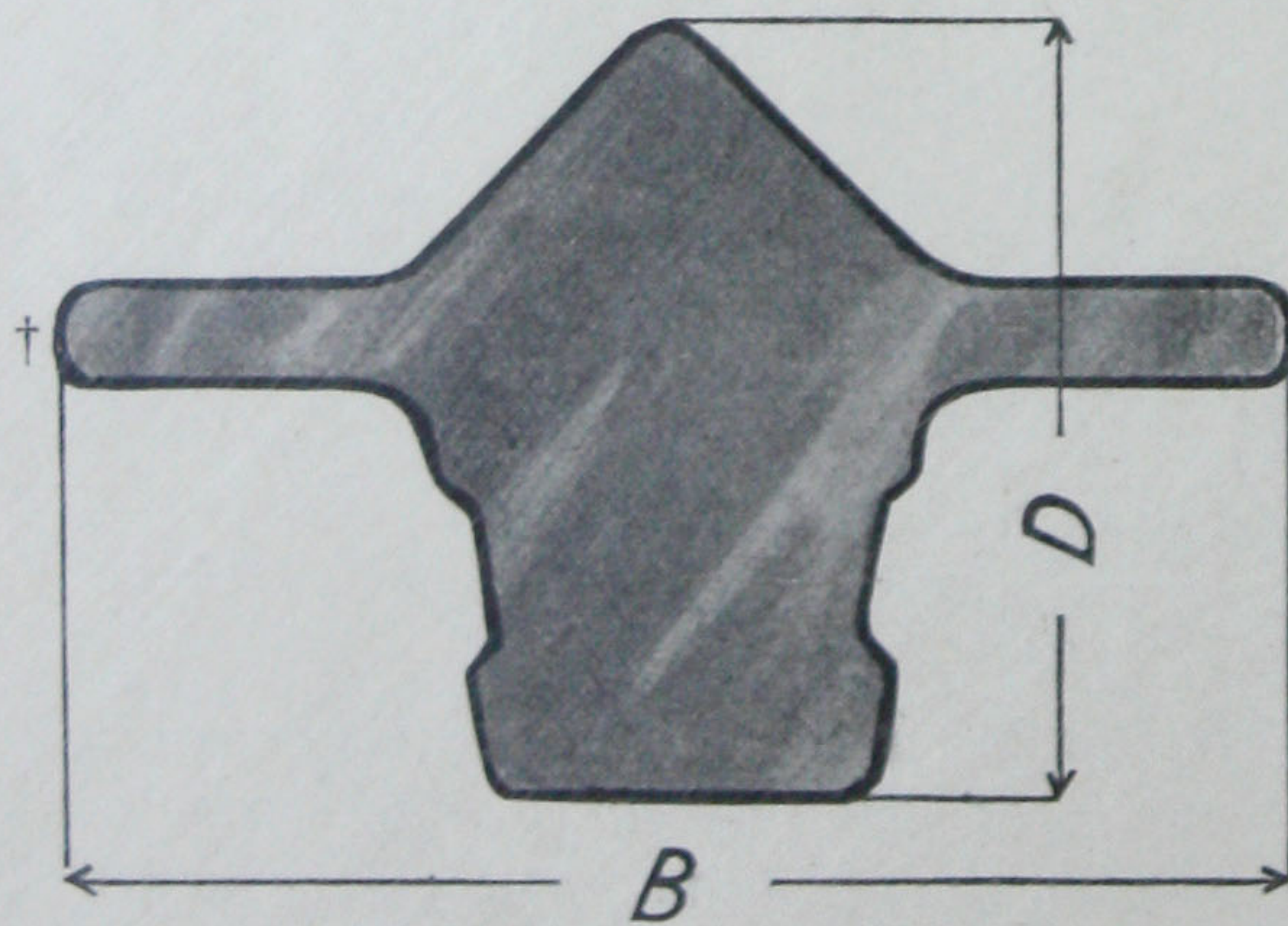
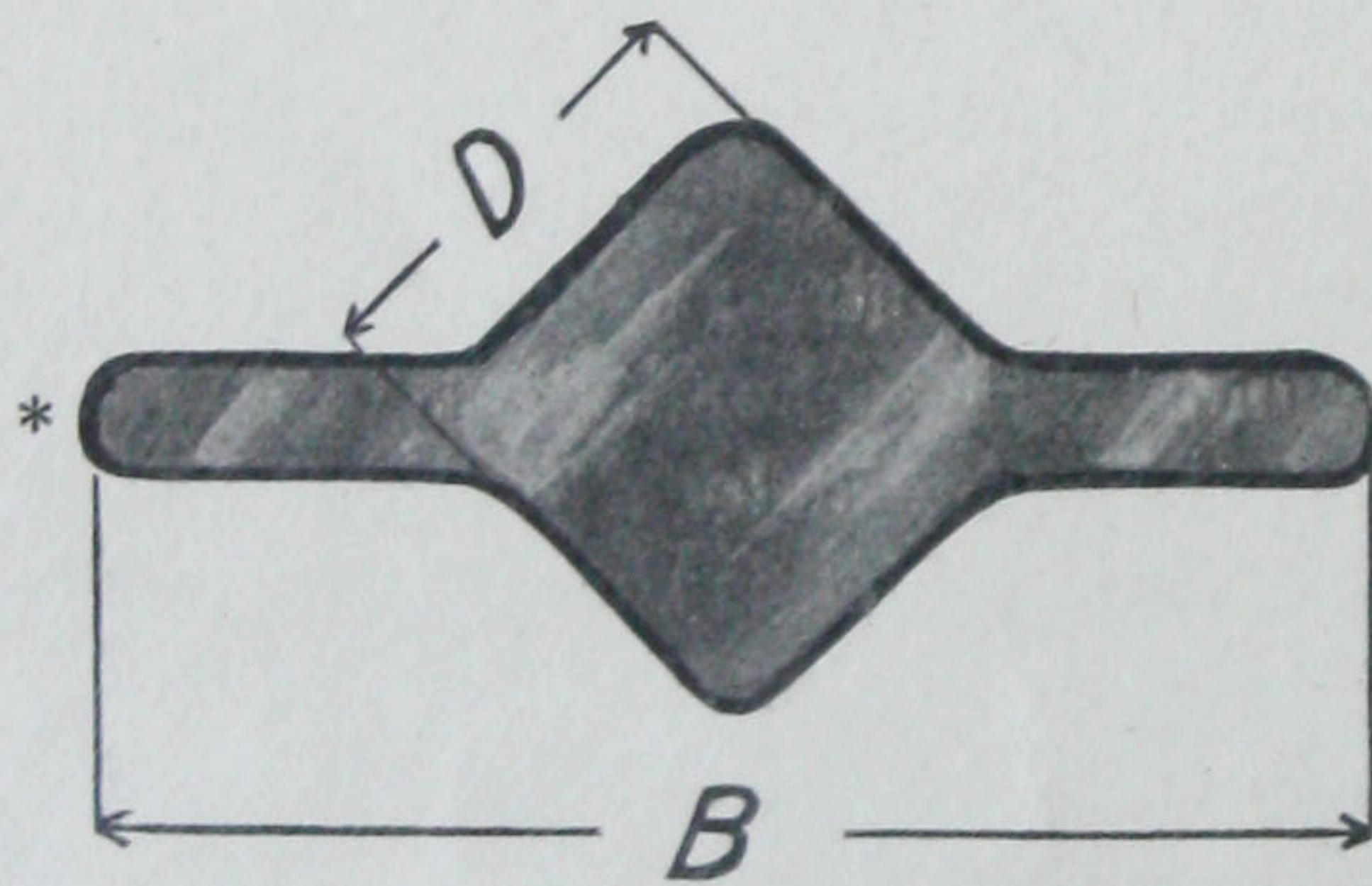
Figure "Y"

*See page 38, Proceeding of the 15th Annual Convention American Concrete Institute.



KAHN TRUSSED BARS

Size D. x B.	Nominal Size	Weight Per Ft.	Area Sq. Ins.		Perimeter		Nesting Capacity		Prongs		Max. Length
			Gross	Net	Gross	Net	Kahn Bar	Rib Bar	Std.	Spec.	
* $\frac{1}{2}" \times 1\frac{1}{2}"$	$\frac{1}{2}"$	1.4 lbs.	0.41	0.25	3.62"	2.12"	None	$\frac{3}{4}"$ rd. Max.	12"	8"-18"	60'-0"
* $\frac{3}{4}" \times 2\frac{3}{16}"$	$\frac{3}{4}"$	2.7 lbs.	0.79	0.56	5.37"	2.87"	$\frac{1}{2}"$ Max.	1" rd. Max.	12-24-36	8"-30"	60'-0"
† $1\frac{1}{2}" \times 2\frac{1}{4}"$	$2\frac{1}{4}"$	4.8 lbs.	1.41	1.10	6.50"	4.00"	$\frac{3}{4}"$ Max.	1" rd. Max.	12-24-36	8"-30"	60'-0"
† $1\frac{3}{4}" \times 2\frac{3}{4}"$	$2\frac{3}{4}"$	6.8 lbs.	2.00	1.60	8.12"	5.12"	$2\frac{1}{4}"$ Max.	$1\frac{1}{8}"$ sq. Max.	36"	24-30-48	60'-0"
† 2" x $3\frac{1}{2}"$	$3\frac{1}{2}"$	10.2 lbs.	3.00	2.40	9.62"	6.12"	$2\frac{3}{4}"$ Max.	$1\frac{1}{4}"$ sq. Max.	36"	24-30-48	60'-0"



END SHEAR TAKEN BY KAHN BAR PRONGS

Size of Bar	Shear
$\frac{1}{2}" \times 1\frac{1}{2}"$	1800
$\frac{3}{4}" \times 2\frac{3}{16}"$	2600
$1\frac{1}{2}" \times 2\frac{1}{4}"$	3500
$1\frac{3}{4}" \times 2\frac{3}{4}"$	4600
2" x $3\frac{1}{2}"$	6800

CENTER OF GRAVITY OF KAHN AND NESTED RIB BARS

Rib Bar	None	$\frac{3}{8}"$	$\frac{1}{2}"$	$\frac{5}{8}"$	$\frac{3}{4}"$	$\frac{7}{8}"$	1"	$1\frac{1}{8}"$	$1\frac{1}{4}"$
Std. Shear Kahn Bar	$3\frac{1}{2}"$	1.25	1.28	1.32	1.38	1.45	1.53	1.62	1.74
	$2\frac{3}{4}"$	.94	.99	1.05	1.12	1.21	1.32	1.43	1.56
	$2\frac{1}{4}"$	.81	.88	.95	1.04	1.14	1.25		
	$\frac{3}{4}"$	.47	.55	.63	.74	.85	.97		
	$\frac{1}{2}"$	.32	.44	.56	.68				
Center Shear Kahn Bar	$3\frac{1}{2}"$	1.23	1.27	1.32	1.39	1.47	1.57	1.68	1.81
	$2\frac{3}{4}"$	.89	.96	1.03	1.12	1.23	1.36	1.48	1.62
	$2\frac{1}{4}"$	.76	.86	.94	1.05	1.17	1.29		
	$\frac{3}{4}"$	.47	.58	.68	.80	.93	1.05		
	$\frac{1}{2}"$	.32	.49	.64	.76				

Dimensions in inches from bottom face of Kahn Bar up.
Measured from actual nesting.

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